scale. The choice of coordinates was based on the assumption that there is a linear dependence of resistive forces on material density, which was noted for the average values of the forces [4]. It was further assumed that fluctuations of the forces produced by displacement of the solid phase during motion of gas bubbles is determined by the filtration of an excess amount of gas above that needed to maintain the material in a suspended state. In the assumed coordinate system, the experimental points obtained at various air filtration rates for five different materials are grouped around a straight line described by the relation

$$\frac{G}{\rho} = (U - U_0)^{0.66}.$$

The greatest deviation of experimental points from the approximating relation does not exceed 70%, which must be considered satisfactory for so unstable a system as a fluidized bed. It must be pointed out that this relation also extends to experiments with large particles where the resultant gas bubbles become commensurate with the cross section of the column and plunger displacement of the material in the bed is observed.

The relation obtained demonstrates the effect of material characteristics and of gas filtration rate on maximum forces in a bed, but it does not reflect the effect of the geometric parameters of the system.

NOTATION

d, particle diameter; G, force acting on a body in a fluidized bed; U_0, rate for initiation of fluidization; U, gas filtration rate; ρ, material density; R, bubble radius.

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FRAMEWORK CONDUCTION IN A GRANULAR SYSTEM

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Equations are derived for the effective transport coefficients in a system of contacting spherical particles immersed in a nonconducting medium.

A substantial contribution can come from the contacting-particle framework to the transport processes in a high-concentration granular medium; for instance, this framework component can have a marked effect on the total heat flux in a granular medium if the thermal conductivity of the particles is much higher than that of the continuous phase (see [1, 2] for a survey of the experimental data). In particular, the theory of thermal conductivity for granular materials [3] for $\lambda_1 \gg \lambda_0$ always gives results for the effective thermal conductivity systematically lower than those from experiment if the transfer by contact between the particles is neglected, whereas theory agrees extremely well with experiment if $\lambda_1 \leqslant \lambda_0$.

Under certain extreme conditions, this component of the flux may be the dominant one. For instance, it has been found [4] that this occurs for uranium and zirconium powders in various gases at pressures below 10^{-2}–10^{-1} mm Hg.

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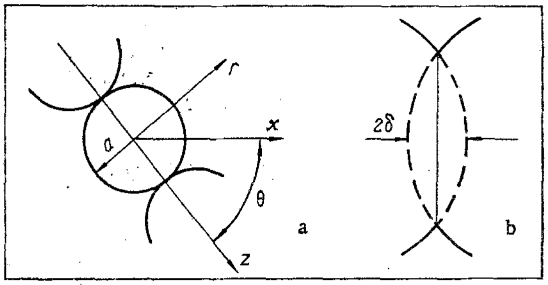

Fig. 1. Particle in contact with adjacent particles (a) and geometry of a single contact (b).

The effects of particle contacts and the framework conductivity are even more substantial in electrical conduction in an immobile granular material with permanent contacts between particles and also in a fluidized bed where the particles are in contact as a result of collision if the continuous phase is an insulating medium or if the electrical conductivity of the latter is much less than that of the particles [5].

The existing treatments of framework conduction are very similar and involve either approximate simulation in terms of conducting cells of variable cross section [2] or numerical examination of the boundary-value problem for the Laplace equation near a single contact [6, 7]. In what follows, the effective framework conductivity of an immobile granular bed is considered via a simple ensemble-averaging procedure. The treatment is simplified by considering the continuous phase as completely nonconducting, while the particles are treated as spheres of identical radius a. As a rule, heat transport is envisaged, although the results apply equally to the transport of any analogous quantity.

The particles are in random contact, the number of contacts varying from one particle to another, as do the positions of the contacts with respect to the laboratory axes, which pass through the center of a particle. That is, the number of contacts is random and the contacts themselves are specified by random angular variables and have random microphysical parameters. If we average over many particles under identical macroscopic conditions (i. e., an ensemble of particles), we arrive in the usual way at a concept of particles subject to certain average microscopic conditions (i. e., trial particles). The contacts of such a particle with its neighbors may be characterized in terms of a distribution, which is denoted in what follows by $\Phi(\theta, \varphi)$ in terms of the angular variables θ and φ; this function is normalized to the coordination number ξ of the particles in the bed. Contacts with given θ and φ are characterized by means of certain average microphysical parameters, which are single-valued functions of these angular variables. The distribution is dependent not only on the parameters, but also on the quantities that describe the macroscopic state of an element in the bed.

There is a direct method of deriving the relationship between the mean heat flux and the mean temperature gradient in the system by solving the thermal-conduction problem for a single particle with a specified number of contacts of specified disposition and parameters, subject to the condition that there is no heat loss over the surface of a particle apart from the contact areas, where different boundary conditions apply. This approach allows us to derive a linear relationship between the mean temperature gradient and the mean heat flux after averaging over the volume of a particle. The subsequent averaging over the ensemble in principle results in the desired equation.

This traditional approach is very difficult to realize on account of difficulties in formulating the boundary conditions for the contact areas and in solving the boundary-value problem for the thermal-conduction equation, since averaging over the ensemble with respect to $\Phi(\theta, \varphi)$ involves integration over a complicated configuration space formed by the angular variables for all the contacts. A different and much simpler approach is therefore used below.

First, the averaging over the ensemble is performed, and then the thermal-conduction problem is solved for a particle whose contacts with other particles are described by $\Phi(\theta, \varphi)$. It is clear that this approach is justified because the above operations can be reversed in sequence, which itself follows directly from the commutation of ensemble averaging and differentiation, as well as from the linearity of thermal conduction. This feature allows one to overcome the above difficulties and to reduce the initial extremely complicated problem to a series of elementary ones.

The averaging is based on the observation that the macroscopic properties of a granular bed scarcely vary over distances of the order of the microscopic scale, i. e., the sizes of the individual grains (this, in general, is a necessary condition for one to use continuum methods in describing any transport process in a heterogeneous medium). This means that the statistical weights and microphysical parameters of opposite contacts on the trial particle may be considered as identical, so we have a trial particle whose surface has a continuously distributed pair of identical diametrically opposite contacts. † The statistical weight of this pair is described by $f(\theta, \varphi) = {}^1\!/_2\, \Phi(\theta, \varphi)$, which is clearly normalized to $\xi/2$.

We first consider the heat transport in a particle due to heat transfer through the contacts with adjacent particles as in Fig. 1a; the line joining the centers of the contact areas forms an angle θ with the mean heat-flux direction in the bed (the x axis). Each contact has an area s or an indentation distance δ (Fig. 1b), which are related‡ by

$$s = 2\pi a\delta \quad (\delta \ll a). \tag{1}$$

In general, s and δ are dependent on the orientation of the contact with respect to the principal axes of the state of strain in the bed (i. e., they are functions of θ and φ). Since there is no heat transport in the gaps between the particles, the vectors representing the heat flux start from one contact area and pass to the other, while remaining entirely within the particle; they also are tangential to the surface at all points, apart from the points of contact.

We introduce a cylindrical coordinate system whose axes z and r are shown in Fig. 1a; we integrate the local relation $\mathbf{Q} = -\lambda_1 \nabla T$ over the section of a sphere by a plane normal to the z axis at the point having coordinate z to get

$$q_z^* = -\lambda_1 \pi (a^2 - z^2) \langle \nabla T \rangle_z^* (z). \tag{2}$$

Clearly, the heat flux q_z^* through these contact areas is independent of z, while the mean value of the z component of the temperature gradient is dependent on the position.

We average the temperature gradient over the volume of the particle, for which purpose the quantity $\langle \nabla T \rangle_z^*(z)$ appearing in (2) must be averaged with respect to z over the range $-(a-\delta)$ to $a-\delta$; we use the symmetry, with respect to the plane z = 0 and take only the principal term in the expansion with respect to the small quantity δ/a to get

$$\langle \nabla T \rangle_z^* = -\frac{q_z^*}{\lambda_1 \pi (a-\delta)} \int\limits_0^{a-\delta} \frac{dz}{a^2 - z^2} \approx -\frac{q_z^*}{2\lambda_1 \pi a^2} \ln \frac{2a}{\delta}. \tag{3}$$

Of course, the flux q_z^* may be dependent on the orientation of the pair of contacts (i. e., on θ and φ), but this is unimportant for the extraction from the integral with respect to dz in (3). Then (1) and (3) give us an expression for the flux:

$$q_z^* = -\frac{2\pi a^2 \lambda_1}{\ln(1/\nu)} \langle \nabla T \rangle_z^*, \quad \nu = \frac{s}{4\pi a^2} \ll 1, \tag{4}$$

where the mean temperature gradient in a particle appears on the right, while ν represents the contact area as a fraction of the total surface area.

The quantities q_z^* and $\langle \nabla T \rangle_z^*$ in (4) relate to the temperature distribution due solely to the two opposite contacts; we now derive the relation between the analogous quantities due to all the contacts of the particle with its neighbors. It is clear that the above with the definition of $f(\theta, \varphi)$ and the linearity of thermal conduction together imply

$$q_x^* = \int\limits_0^{2\pi} d\varphi \int\limits_0^{\pi} d\theta q_z^* \cos\theta f(\theta, \varphi). \tag{5}$$

Further, the quantity $\langle \nabla T \rangle_z^*$ in (4) is equal to the mean temperature difference between the contacts along the z axis as divided by the particle diameter $2a$; Fig. 1a shows that this difference is equal to the true mean temperature gradient $\langle \nabla T \rangle_x^*$ along the x direction for the mean heat flux in the system as multiplied by the

† Here, of course, we envisage random packing; if the packing is regular, the distribution is not continuous, but in that case the problem is even simpler.

‡ The contact geometry shown in Fig. 1b and implied by (1) corresponds to idealization of an actual contact, since no allowance is made from the local deviation from spherical shape.

projection on the x axis of the diameter of the sphere parallel to the z axis, i. e., by $2a\cos\theta$, so $\langle\nabla T\rangle_z^* = \langle\nabla T\rangle_x^* \cos\theta$, and (4) and (5) give

$$q_x^* = -2\pi a^2\lambda_1 \langle \nabla T \rangle_x^* \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \frac{\cos^2\theta f(\theta, \varphi)}{\ln[1/\nu(\theta, \varphi)]}. \tag{6}$$

In particular, the conditions of uniform state of strain (i. e., complete statistical isotropy in the bed) must imply that $\nu(\theta, \varphi) = \nu = \text{const}$ and $f(\theta, \varphi) = \zeta/8\pi$, so from (6) we have

$$q^* = -\frac{\zeta\pi^2 a^2}{4}\lambda_1 \ln^{-1}(1/\nu)\langle \nabla T \rangle^*. \tag{7}$$

In the general anisotropic case, the dependence on the angular variables is substantial, and the coefficient of proportionality between $\langle\nabla T\rangle_x^*$ and q_x^* in (6) will be dependent on the direction of heat propagation, so the choice of this direction influences the definition of the angular variables and hence that of the $\nu(\theta, \varphi)$ and $f(\theta, \varphi)$ functions, so (7) is replaced by the more general formula

$$\mathbf{q}^* = -\frac{\zeta\pi^2 a^2}{4}\lambda_1 \mathbf{N} \langle \nabla T \rangle^*, \tag{8}$$

where $\mathbf{q}^*$ and $\langle\nabla T\rangle^*$ are vectors, so $\mathbf{N}$ is a true second-rank tensor. It is clear that isotropic packing (in particular, chaotic or random) implies that the principal axes of this coordination tensor, which render the tensor diagonal, should coincide with the principal axes x_i of the state of strain, while the eigenvalues N_i can be calculated from (6) by setting the x direction along the corresponding principal axis x_i. It is convenient to represent the eigenvalues in the form

$$N_i = \ln^{-1}(1/\nu_i) = -\ln^{-1}\nu_i \ (i = 1,\ 2,\ 3), \tag{9}$$

where ν_i has the meaning of the effective fraction of the contact areas along the direction i. The ν_i can be expressed in terms of the parameters that govern the distribution and properties of the contacts by means of (6). In certain instances (particularly for beds of rough or irregular particles, where no rigorous analysis is possible) it is convenient to consider these quantities as empirical parameters to be determined, for instance, from experiment.

We now derive the relation between the mean heat flux $\mathbf{q}$ and the mean temperature gradient $\nabla_T = \nabla\langle T\rangle$ for the granular bed as a whole; first of all, we consider lines that join any two points on the particles and that lie entirely in the dispersed phase, which gives $\nabla\langle T\rangle \equiv \langle\nabla T\rangle$; further, $\mathbf{q}$ is equal to the product of $\mathbf{q}^*$ by the mean number of particles that intersect unit area normal to the x axis. The latter is the result from dividing ρ by the mean area of intersection between unit sphere and such an area, which is $2\pi a^2/3$, so (8) finally gives

$$\mathbf{q} = -\Lambda\nabla\tau,\ \Lambda = {}^3/_8\, \pi\zeta\rho\lambda_1\mathbf{N}, \tag{10}$$

where the eigenvalues Λ_i of the tensor Λ act as thermal conductivities along the axes x_i.

Equations (9) and (10) relate the effective framework conductivities Λ_i to the conductivity of the particles and the packing characteristics; they provide major conclusions on the effects of various observable quantities on the transport. For instance, we may examine the effects on Λ_i from the corresponding principal normal stress σ_i and the particle size, for which purpose we express ν_i in terms of these quantities by means of the solution to the corresponding contact problem in the theory of elasticity (a Hertz problem in the present case). This solution implies [8] that

$$\nu_i \sim F_i^{2/3}E^{-2/3}a^{-4/3}, \tag{11}$$

where the force F_i acting on a single contact in direction i is proportional to $\sigma_i a^2$, and so

$$\frac{1}{\nu_i} = C\left(\frac{E}{\sigma_i}\right)^{2/3},\ \ln\frac{1}{\nu_i} = \ln C + \frac{2}{3}\ln\frac{E}{\sigma_i} \approx \frac{2}{3}\ln\frac{E}{\sigma_i}, \tag{12}$$

where C is a coefficient of proportionality of the order of 1, which in most instances can simply be neglected. Then (12) illustrates the relationship of N_i and Λ_i from (9) and (10) to the corresponding compressive stress σ_i. The state of stress in a real bed is usually anisotropic, so when we speak of framework conductivity the bed is to be considered as a body with anisotropic thermal or electrical conduction.

Also, (12) illustrates how the framework conduction is dependent on the speed of an ascending flow of continuous phase passing through the bed. If we neglect the resistance, which does not vanish even if the weight of the particles is balanced by the upthrust, the stresses $\sigma_i(u)$ can be described approximately as follows:

$$\sigma_i(u) = \frac{\gamma - \psi(u)}{\gamma}\,\sigma_i(0),\quad \psi(u) \sim u^{\varkappa}, \tag{13}$$

where $\varkappa = 2$ and $\varkappa = 1$ for the limiting cases of large and small Reynolds numbers for a single particle, respectively. The stresses become zero when the speed reaches the value corresponding to onset of fluidization u_0, which satisfies $\psi(u_0) = \gamma$; a simple calculation from the above formulas gives

$$\Lambda_i = \frac{A}{B_i - 2.3\ln\left[1 - (u/u_0)^{\varkappa}\right]},\qquad A = \frac{3}{8}\pi\zeta\rho\lambda_1,\quad B_i = \frac{2}{3}\ln\frac{E}{\sigma_i(0)} - \ln C. \tag{14}$$

This implies that the Λ_i decrease monotonically as u increases, and the more rapidly the smaller $\varkappa$, i. e., the smaller the particles. This decrease is of logarithmic type, and it is not particularly large if u is not too close to u_0, but the Λ_i rapidly become zero* as u tends to u_0. The dependence of u is the more pronounced the smaller B_i in (14), i. e., the higher the initial stresses in the bed. In particular, the form of the relationship tends to vary with the level in the bed, and these general conclusions are confirmed by experiments on the electrical conductivity of beds of steel spheres [9].

The arguments leading to (11)-(14) deal only with the regions of direct (physical) contact between the particles, which is sufficient for the electrical conductivity of a bed in an ideal insulating medium and also for heat transport in a granular bed at very low pressures or very low temperatures.

It also follows from (12) that the framework-conduction coefficient should not be dependent on the particle radius. †

The contribution from the framework conduction can be estimated by putting the flux of (10) into correspondence with the flux due to the conductivity in the continuous phase, provided a correction is applied for the distorted isotherms and flow lines. If we use the result of [3] for this flux, we find that the contacts are important in the transport if ν satisfies the inequality

$$\ln\frac{1}{\nu} < \frac{8\pi\zeta\rho}{3\beta(\rho)}\,\frac{\lambda_1}{\lambda_0}, \tag{15}$$

where $\beta(\rho)$ is the ratio of the effective conductivity of the bed on neglecting the contacts between the particles to the conductivity of the continuous phase when $\lambda_1 \gg \lambda_0$; for the values of ρ of interest given in [3] we have $\beta(\rho) \sim 10$.

In the general case, the ν_i are dependent not only on the stresses in the bed, but also on the parameters representing the continuous phase and the rate of the transport [2]; for instance, the effective contact areas increase somewhat in heat transfer at low gas pressures on account of the annular regions near the direct contacts in which the distances between the surfaces are comparable with the mean free path of the gas molecules, in which case there is heat transfer by the free-molecule mechanism. Radiative heat transfer can result in the same effect, and this is important at high temperatures. Therefore, the effective areas of contact may exceed substantially the areas of physical contact.

If an electrical current flows in such a bed (charge transport), electrical breakdown can occur in the narrow gas gaps separating the particles, and therefore the ν_i and the effective conductivity will increase with the current, as has been observed [5]. The same applies to any other change in the external conditions that facilitates ionization of the gas between the particles, e. g., the specific resistance of the bed should fall as the gas humidity and temperature are increased, as has been observed [10]. The available measurements on the effects of temperature, current, and other factors on the electrical conductivities of granular beds indicate that this theoretical model provides a good qualitative description. For instance, one expects that the effects of the current will be less at high temperatures, since the conditions are already favorable to ionization; further

* In fact, of course, the effective conductivities do not become exactly zero, because some contacts persist in the bed even if $u = u_0$, and the number of these is the larger the greater the deviation from ideal spherical form. Some part is played also by the resistive stresses, which do not vanish for $u = u_0$.

† Of course, this argument neglects the possibility that the type of packing (e. g., the coordination number) is dependent on the particle size for a given macroscopic state of stress.

the electrical conductivity will be dependent on the ionization potential of a gas more markedly the lower the humidity and temperature, and so on.

So far, the particles have been considered as identical, while the thermal conductivity of the continuous phase has been taken as close to zero; if we wish to extend the theory to a situation where even one of these assumptions does not apply, it becomes necessary to solve several different independent problems, and the treatment falls outside the framework of this study. Here we merely briefly note some major features that may occur.

We first assume that the conduction in the continuous phase is comparable with the contact conduction, i. e., the quantities in (15) are comparable; in that case we can use the general method of [3] but with independent heat transport for the dispersed phase. As a result, the effective thermophysical parameters of the dispersed medium are again determined by solving the problem for a trial particle, but the treatment is then more complicated than that of [3] because the temperatures of the phases cannot be considered as identical even under steady-state conditions, so it is necessary to consider heat transfer from a trial particle not to one fictitious homogeneous medium but to two such, which stimulate the individual phases and which are represented by different transport equations.

Let the bed consist of spherical particles of different sizes but composed of the same material; if we neglect the heat transport through the continuous phase, the problem is readily reduced to that examined above provided we introduce the distribution $f'(a)$ for the radii of the spheres and the distribution $f''(a; a')$ for the radii of the particles a in contact with the particle of radius a'; on the simplest assumption (an entirely random structure) we have $f''(a; a') = f'(a)$. In that case, the properties of the contacts will be dependent on the radii of both contacting particles [the relation replacing (1) is readily derived], and instead of $f(\theta, \varphi)$ we have to consider the distribution $f(\theta, \varphi; a, a')$ for the contacts of particles of radius a' with other particles of radius a such that

$$\iint f(\theta,\ \varphi;\ a,\ a')\,d\theta d\varphi = \frac{\zeta}{2} f''(a;\ a'),$$
$$\int f(\theta,\ \varphi;\ a,\ a')\,da = f(\theta,\ \varphi;\ a'),\ \int f(\theta,\ \varphi;\ a')\,da' = f(\theta,\ \varphi), \tag{16}$$

where $f(\theta, \varphi; a')$ is the distribution of the pairs of contacts for particles of radius a' with particles of any radius, with a' treated as a parameter. A simple argument completely analogous to that above gives us

$$q_x^* = 2\pi\lambda_1 \langle \nabla T \rangle_x^* \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int da \int da' \frac{a'^2 f(\theta,\ \varphi;\ a,\ a')\cos^2\theta}{\ln[1/\nu(\theta,\ \varphi;\ a,\ a')]}, \tag{17}$$

which replaces (6); the transfer from (17) or from equations following from (17) of the type of (7) and (8) to an equation of type of (10) is elementary: it is only necessary to determine correctly the mean area of the intersection between any particle in the bed and a plane by averaging the quantity $2\pi a^2/3$ used in deriving (10) over the distribution $f'(a)$.

It is more complicated to incorporate differences in physical properties between the particles; to illustrate this we consider only the transport in a binary mixture of particles identical in size and such that the transport coefficient for particles of the first kind is λ_1 (different from zero), whereas the value for the particles of the second time is zero. As before, we neglect the transport in the continuous phase and introduce the fraction α of conducting particles. This situation is of direct practical interest, since dilution of conducting particles with inert ones is sometimes used in operations with electrothermal granular beds to increase the specific resistance and thus to reduce the heat release on applying a voltage. In that case, we have to consider the effective conductivity of a network of linked identical resistors that stimulate the particles, with the structure of the network defined by the bed packing features, while there is a probability $1-\alpha$ that particular particles may not be involved in the transport. This is a classical node problem in flow theory (see recent reviews in [11, 12]), which has previously been used for locally inhomogeneous semiconductors, insulators, ferromagnetics, and so on, although the treatment is extremely complicated and requires additional analysis. The problem becomes even more complicated if the particles of both types are of finite conductivity or if there are several types of particle.

NOTATION

A, B_i, parameters in (14); a, particle radius; C, coefficient in (12); E, Young's modulus; F, compression force; f, f', f'', local distribution fluxes; N_i, eigenvalues of coordination tensor $\mathbf{N}$; Q, $\mathbf{q}$, local and mean heat fluxes; s, area of contact; T, temperature; u, u_0, filtration speed and fluidization onset speed; x_i, stress axes; α, proportion of conducting particles in binary mixture; $\beta(\rho)$, ratio of effective conductivity of medium containing noncontacting particles to the effective conductivity of the continuous phase for $\lambda_1 \gg \lambda_0$; γ, particle density minus specific buoyancy; δ, compression length; ζ, coordination number; θ, φ, angular coordinates of contact relative to mean flow direction; $\varkappa$, exponent in (13) and (14); Λ_i, eigenvalues of conductivity tensor Λ; λ_0, λ_1, conductivities of continuous and dispersed phases; ν, fraction of surface area represented by a single contact; ρ, volume content of dispersed phase; σ, compressive stress; τ, mean temperature; Φ, distribution function; $\psi(u)$, hydraulic force per unit particle volume; *, values referred to one particle; $\langle\ \rangle$, averages.

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A COMBINED NUMERICAL METHOD FOR DETERMINING THE CONDUCTANCE OF COMPOSITE BODIES

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We propose a new numerical method (a combination of the method of grids with Rayleigh's method) which is very promising for the calculation of potential fields, fluxes, and conductance of composite bodies, especially in the case of components with sharply differing properties.

We consider a two-component region in the form of a cylinder made up of two hemispheres which are in contact at the point A (Fig. 1). As an example, we consider the problem of determining the effective conductance, say the effective thermal conductivity, of the composite region. We denote the thermal conductivity of the material of the hemispheres by λ_1 and that of the material filling the gap between them by λ_2, where λ_1 and λ_2 may be substantially different. Suppose (for the sake of definiteness) that the bases of the cylinder are isopotential (isothermal) planes and that the lateral surface is impenetrable to the streamlines (an adiabatic surface). Such a composite system is often used for constructing a model of the structure of granular materials when we calculate their effective coefficients of generalized conductance (thermal conductivity, electrical conductivity, dielectric permittivity, magnetic permeability, etc.).

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